GREAT CIRCLE SAILING AND GREAT CIRCLE SAILING APPROXIMATION BY USAGE OF TRIGONOMETRIC FUNCTIONS

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Abstract: *Trigonometry was, as a part of mathematics, until the renaissance almost exclusively usedin astronomy, and not as a mathematical theory. Plane ttrigonometry was used just as much as needed for spheerical trigonometry. There are many uses of trigonometric functions today. One of theese uses is visible in marine, specially in navigation. Numerical calculation of the shortest way between starting position and destination position, ie. great circle, and aproximation methods are the backbone of this paper. Paper does, at least partially, show us huge role of trigonometric functions in great circle sailing, and in marine in general.*

Keywords: *mathematics, trigonometric functions, great circle sailing, approximation.*

1. INTRODUCTION

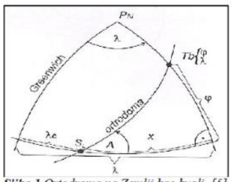
The orthodrome is part of the main circle, and the shortest route between the departure position and the arrival position. In practice, it rarely sails in the orthodrome, because then the helmsman should constantly change course, which is unacceptable in navigation. Only when navigating the orthodrome is when the helmsman manages the vessel by navigating not according to the devices, but towards some marvelous maritime navigation object. All of the above implies that the Earth is a ball. [2, 12] In navigation, the Earth is indeed considered a ball, but for theoretical reasons it is desirable to approximate the Earth's geoid with the rotational ellipsoid [1, 4, 7] that is closer to the Earth's appearance than the ball. If a normal passing through the point Td is laid on the ellipsoid at the point of departure Tp, then this plane cuts the rotation ellipsoid with a normal cross-section at the point Td. However, at point Td this is no longer a normal cross section. Thus, on the rotational ellipsoid, a normal cross-section can not be established by the points Tp and Td, which would mean that these two points can not be withdrawn by the geodetic line as a normal cross-section of both points. The angle between the normal section by the point Tp, passing through the point Td and

meridian, is called the astronomical azimuth of the point Td because it is determined by astronomical observation. The angle between the plane of the daybreak, the point Tp, and the geodetic line by the points Tp and Td of the rotating ellipsoid is called the geodetic azimuth of the point Td. The difference on the rotational ellipsoid of these two azimuths at a distance of 450 km is in the port of 15.43 m, and at distances less than 100 km, it practically disappears. The difference in the distance of the geodetic line by the points Tp and Td and the normal cross sections by either the point Tp (and passing through the point Td) or by the point Td (which passes through the Tp) are so small that they disappear. In this paper numerical calculations of the orthodrome elements and methods of approximation of navigation by orthodrom will be shown.

2. CALCULATION OF THE ORTHODROME SURFACE ELEMENTS

The equation of the orthodrome on the ball is obtained from Fig. 1 according to Nepier's rule [6]:

$$\begin{aligned} \sin x &= ctgA \cdot \tan\varphi, \\ \tan\varphi &= \tan A \cdot \sin x, \\ \tan\varphi &= \tan A \cdot \sin\left(\lambda - \lambda_{\check{c}}\right). \end{aligned} \tag{1}$$



Slika 1.Ortodroma na Zemlji kao kugli, [5]

Since the orthodrome is part of the main circle, the two-point connector on Earth is closer to the earth's poles than the rhumb line. The coupling of two points with $\Delta\lambda$ = 180 ° passes through half. When traveling by boat, parts of the land or the eternal ice obstacle are navigating. Similarly, discarded ice pieces - ice dunes and boulders - pose a threat to navigation. In large latitudes, poor meteorological conditions hinder or limit navigation.

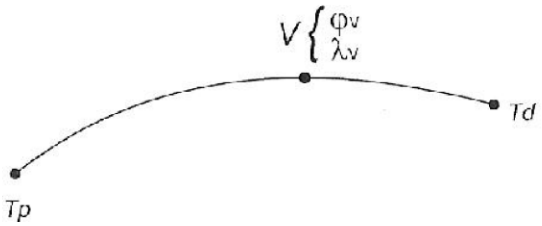


Figure 2.Tip (vertices) of the orthodrome on Earth as a ball, [5]

Therefore, it is of critical importance in choosing the route by which the ship will sail, to determine the maximum latitude to which the orthodrome will lead, which is the vertex orthodrome (Figure 2).
After that, the navigator will study navigation manuals, pilot charts, marine ads, and follow the boat to this orthodrome only if it is not closer to the equator than the forbidden navigation areas. If the orthodrome sets too close to the half of the permitted, it will use combined navigation; will sail part of the road by the orthodrome, part by comparison, and the part will be re-entered by the orthodrom. Whether the boat sails by orthodrome or rhumb lines, depends on how much the orthodrom is shorter than the distance between the point of departure and the point of arrival. The orthodromic distance to (Figure 3) is determined according to the cosine instruction for the sides of the scuffed spherical triangle. [8,9]

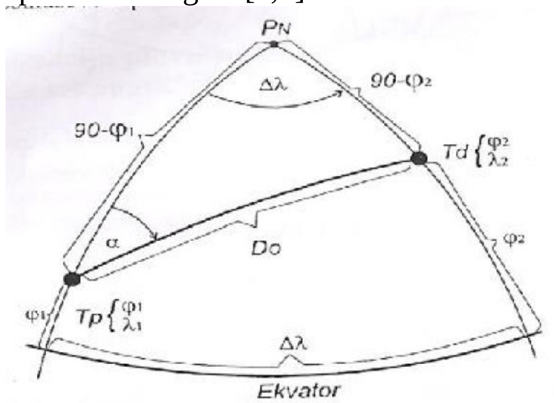


Figure 3.Orthodromic distance on the Earth as a sphere, [5]

$$\begin{aligned} &\cos D_0 = \cos\left(90-\varphi_1\right)\cdot\cos\left(90-\varphi_2\right)+\sin\left(90-\varphi_1\right)\cdot\sin\left(90-\varphi_2\right)\cdot\cos\Delta\lambda \Rightarrow \quad [10] \\ &\Rightarrow \cos D_0 = \sin\varphi_1\cdot\sin\varphi_2+\cos\varphi_1\cdot\cos\varphi_2\cdot\cos\Delta\lambda. \end{aligned} \tag{2}$$

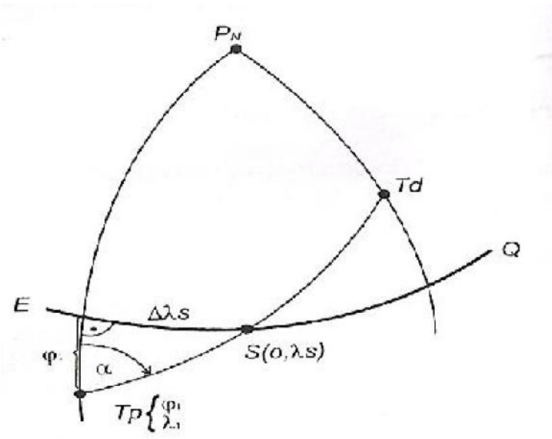
The initial Kpč course is defined by an orthogonal angle α. The orthodromic angle α is determined from the orthodromic triangle shown in Figure 3, according to the sinus instruction [11]

$$\sin\alpha : \sin\Delta\lambda = \sin\left(90-\varphi_2\right) : \sin D_0 \Rightarrow \sin\alpha = \frac{\sin\Delta\lambda\cdot\cos\varphi_2}{\sin D_0} \Rightarrow \alpha = \arcsin\frac{\sin\Delta\lambda\cdot\cos\varphi_2}{\sin D_0}. \tag{3}$$

According to this great circle course the ship usually does not start the journey. If the helmsman kept the course for a long time (then sailed along a rhumb line with an initial orthodromic course), he would move on the tangents at the orthodrom, which would lead him to excessive latitude. The ship will sail along the loxodrorus tendon, and then follow the orthodrome from the intersection to intersection. [3] From a rectangular triangle whose one cathete is a daybreak of the point of departure, the other half of this to the intersection of the orthodrome with the hemisphere, and the hypotenuse of the arc of the orthodrome, is as follows:

$$\cos\left(90-\varphi_1\right) = ctg\left(90-\Delta\lambda_S\right)ctg\alpha \Rightarrow$$
$$\Rightarrow \sin\varphi_1 = tg\Delta\lambda_S ctg\alpha \Rightarrow \Delta\lambda_S = arctg\left(\sin\varphi_1 \cdot tg\alpha\right). \text{[10]}$$

Figure 4. Crossing of orthodrome with the equator, [5]

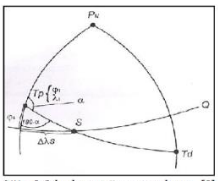


If the position of the orthodrome is in Figure 4, then:

$$\cos\left(90-\varphi_1\right) = ctg\left(180-\alpha\right)\cdot ctg\left(90-\Delta\lambda_S\right) \Rightarrow \sin\varphi_1 = -ctg\alpha \cdot \tan\Delta\lambda_S.$$
$$\tan\Delta\lambda_s = -tg\alpha \cdot \sin\varphi_1 \Rightarrow \tan\Delta\lambda_s = \arctan\left(-\tan\alpha \cdot \sin\varphi_1\right). \quad (5)$$

Geographical length of point S (orthodrome node) will be:

$$\lambda_S = \lambda_1 + \Delta\lambda_S.$$
$$\lambda_{\gamma} + \lambda_{-} = 180°. \quad (6)$$



Slika 5.Određivanje čvora ortodrome, [5]

3. APPROXIMATION OF SAILING ALONG THE ORTHODROME

Considering the fact that orthodromic navigation has been postponed so that in practice it is practiced just like that, and if there is a higher fuel cost if it is sailed by the marathon, special navigation cases have been developed, which is based on the approximation of sailing along the orthodrom. The goal of approximation of navigation by the orthodrome is to save time and fuel at least approximately to that achieved by navigating through a pure orthodrome, and at the same time, to facilitate the orthodromic sailing to the helmsman so that the course should not be constantly changed.The approximation is done by calculating certain points of contact on the path of the orthodrome, and the ship between the same sail along the rhumb line, therefore, without the change of course.The most common methods of approximation of navigation by orthodrome are the method of approximation by secants, and the method of approximation by means of tangents.

3.1. Approximation with the secant method

Approximation by the method of secants is carried out in such a way that through the orthodrome a secant passes through arbitrarily selected intermediate points, and then the ship between the paths sails along a rhumb line.The waypoints of the orthodromes are determined so that their lengths differ by 5 ° or 10 °, and the division starts from the top, i.e. if it is between the position of departure and the arrival position, and if the orthodrome head is located outside the coupling of the position of the departure and the position of arrival, the orthodromes can be defined from the departure position.

If the division starts from the scalar, the relations are derived from Figure 6. Namely, let the geographical length difference be sought for which the intermediate points ψ are sought, and the T-point of the day is given by the coordinates φT and λT. [3] The navigator selects the geographic length of the intersection, i.e.

$$\lambda_T = \lambda_V - \psi. \tag{7}$$

Latitude obtained from a right triangle ΔPNTV [11]:

$$\cos\psi = ctg\left[90-\left(90-\varphi_V\right)\right]ctg\left(90-\varphi_T\right) \Rightarrow \cos\psi = ctg\varphi_V \cdot \tan\varphi_T \Rightarrow \tan\varphi_T = \tan\varphi_V \cdot \cos\psi \Rightarrow$$
$$\Rightarrow \varphi_T = \arctan\left[\tan\varphi_V \cdot \cos\psi\right]. \tag{8}$$

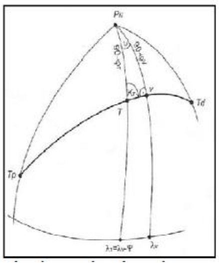


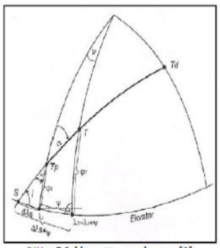
Figure 6. Determination of the coordinates of the orthodrome by the method of the secant, [5]

If the orthodrome has no scalp, it is necessary to first determine the inclusion of the orthodrome according to the equator. Inclination is the angle under which the orthotorm shaves the earth's equator, and is derived from the rectangular triangle of the point of departure. According to Napier's rule [6]:

$$\cos i = \sin\left(90-\varphi_1\right)\sin\alpha \Rightarrow \cos i = \cos\varphi_1 \sin\alpha \Rightarrow i = \arcsin\left(\cos\varphi_1 \sin\alpha\right). \tag{9}$$

If the coordinate λs of the orthodrome cross-section with the equator S is defined earlier, and in the rectangular triangle ΔSAT the known catheter at the equator (Δλs + ψ) and the inclination angle i; from the ΔSAT triangle follows:

$$\cos\left[90-\left(\Delta\lambda_S+\psi\right)\right] = ctgi \cdot ctg\left(90-\varphi_T\right) \Rightarrow \sin\left(\Delta\lambda_S+\psi\right) = ctgi \cdot \tan\varphi_T \Rightarrow \tan\varphi_T = \sin\left(\Delta\lambda_S+\psi\right)\cdot\tan i \Rightarrow$$
$$\Rightarrow \varphi_T = \arctan\left[\sin\left(\Delta\lambda_S+\psi\right)\cdot\tan i\right]. \tag{10}$$



Slika 7.Inklinacija ortodrome, [5]

Other Waypoint great circle shall be determined for every 5 (or 10) degrees to the left or right of the scalp, and the advancement of the point of departure (when there is no great circle vertex) on the same terms as above, only the angle ψ every time increases. Lochschord intermediate is calculated from the loxodromic triangle [11]

$$\tan K = \frac{\Delta\lambda}{\Delta\varphi_M} \Rightarrow K = \arctan\frac{\Delta\lambda}{\Delta\varphi_M}. \tag{11}$$

The first course K1, after which the navigation will be started at the orthodrom at the point Tp (Fig. 8), will be obtained on the basis of the Δλ point of departure and the T1 point of the difference between Mercator's widths of these two points. The second course K2 at the point T1 will be obtained on the basis of analog Δλ and ΔφM points T1 and T2 and thus all courses in a row.

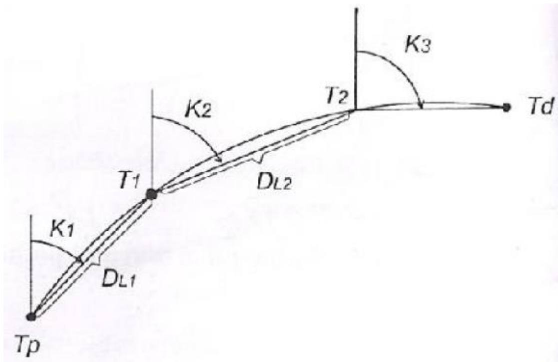


Figure 8. Approximation of orthodrome navigation by secant method, [5]

In order for a navigator to know when a course change is needed in a course in individual points of view, or the time it will arrive at a particular point of departure, the locomotor distance should be determined. It follows from the loxodromic triangle

$$D_L = \frac{\Delta\varphi}{\cos K}. \tag{12}$$

In this case, the DL should be determined Δφ points T1 and Tp and the course K1, for DL2 points T2 and T1 and so on. [3]

3.2. Approximation by tangent method

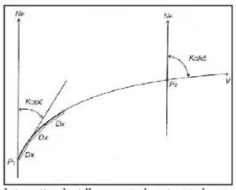
In this case, instead of the secant which are determined by waypoints, navigating to the tangent great circle which is determined as follows: Calculate the initial great circle course at an initial position (P1) - Kopc and final orthodromic course to the destination position (P2) - Kokc, and the following values are calculated:

$$x = \frac{K_{okč} - K_{opč}}{\Delta K} \quad \text{i} \quad Dx = \frac{D_0}{x}[M].$$

$$\text{za} \quad \Delta K = 1^\circ \Rightarrow Dx = \frac{D_0}{K_{okč} - K_{opč}}, \tag{13}$$

gdje je:

- X – ukupna vrijednost promjene ortodromskog kursa,
- ΔK - 1°, 2°, 3°, ... po volji odabrana jedinična vrijednost promjene ortodromskog kursa,
- D_0 – ortodromska udaljenost u nautičkim miljama između dviju odabranih pozicija (P_1) i (P_2) na ortodromi,
- $K_{opč}$– početni ortodromski kurs u ishodišnoj poziciji (P_1),
- $K_{okč}$ – završni ortodromski kurs u odredišnoj poziciji (P_2) i
- Dx – „jedinična ortodromska“ udaljenost u nautičkim miljama koju brod treba prijeći u određenom kursu da bi nastala promjena u ortodromskom kursu od ΔK stupnjeva.



Slika 9.Aproksimacija plovidbe po ortodromi metodom tangenata, [5]

4. CONCLUSION

Seafaring, as a branch of the world economy, is the most cost-effective form of transporting people and goods over long distances. With the development of maritime transport, and the introduction of new technologies in maritime transport, the maritime world's share of the world economy is increasing every day, but at the same time it improves seafaring as such. Throughout history, people have changed travel routes and ways in which ships sail in accordance with the scientific discoveries of the time. Today, the most cost-effective way of navigation is to navigate the orthodrome. Orthodrome sailing greatly disturbs the distance, and saves time and money to shipping companies. Horticultural navigation in practice is not entirely feasible because it would require a constant change in the course of the ship, and it depends on the weather, geological and other conditions of the sea in which the ship sails. Therefore, orthodromic navigation is approximated. By approximating, the paths to the nearest orthodrome are obtained, and significant savings are achieved in relation to the rossoceanic sailing. The approximation of orthodromic navigation is done by mathematical calculation of elements through trigonometric functions. Trigonometric functions are used in every segment of orthodromic navigation, starting from the calculation of courses and distances, by calculating geographical coordinates of positions and intermediate positions, to various methods of approximate orthodromic navigation. Also, trigonometry is also used in the development of navigation maps and electronic navigation systems. Given the widespread use of trigonometric functions in Orthodromic navigation, the same is not conceivable without trigonometry. Trigonometry and trigonometric functions are closely related to maritime and orthodox sailing, and the development of maritime and navigation largely depends on the further development of trigonometry.

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PARENT EDUCATION AND TEACHER AS A FACTOR ROAD SAFETY

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Abstract: *This paper is particularly emphasized another aspect is very important for the formation of attitudes in traffic and the development of personality and its relationship to traffic starting from children as the most vulnerable participants in traffic to the parents and teachers of children who mainly learn the most and depend on in life. Education of all groups of traffic participants that reason is an important part of traffic safety which unfortunately in our country is still not understood seriously enough, nor indeed the mere possibility of traffic management. The paper analyzes the quality of education of children, parents and teachers to acquire knowledge of the traffic on the knowledge of traffic and identification of measures that are important for the preservation of life in traffic. Based on the research have been proposed and corresponding measures to improve this aspect of traffic safety because it is clear how education is important in building positive attitudes and increase awareness of the dangers.*

Keywords: *education of parents, children and teachers*