MATHEMATICAL MODEL ON TRANSPORT NETWORKS

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Abstract: A mathematical model describes a system using a series of sets, equations, variables that describe relationships and relationships between them. The system is an abstract entity, which we believe has no interaction with the environment but is isolated and exists as an independent entity. Usually, the system is defined by the mathematical relations between input and output quantities. The variables in the model represent some of the features of the system. They can be input, output, independent, dependent, state variables and random variables. Most often, mathematical functions are used for modeling, and we optimize the parameters by approximating or interpolating a curve. Mathematical modeling is recognized as the process of applying mathematics to a real system for the purpose of verifying the necessary information. It is important to emphasize that modeling does not have to solve the problem, but is likely to shed light on the problem and clarify the observed situation. The application of mathematical model and programming tools aims to reduce the possibility of such problems occurring and solving them if they do occur. The model is a closed transport problem. It is also shown in the software package matlab one transport problem and approximately solved.

Keywords: Transport model, open and closed model, mathematical model

1. Transportation problem

Solving the transportation problem on the transportation network gives you an optimal way of transporting between multiple supply centers and demand centers. The supplier center has its own capacity and the demand center has its own level of demand. For example, a supplier center may be a specific distributor, and the demand center may be the end user or the customer. The transport routes between the two centers have different unit transport rates, and the solution to this problem is to achieve the best possible solution for transport between nodes. In order for the solution of the problem to be optimal, two conditions must be satisfied and the demand on the network must be satisfied and the other is to do so with minimal transport costs.

Of course, first you need to find a decent solution, and then an optimal one. Methods such as: the least cost method and Vogel's approximation method are used to determine the initial solution. The optimal solution is implemented using the relative cost method.

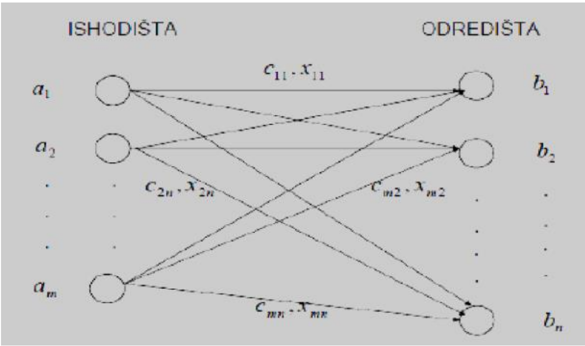

Figure 1. Schematic representation of the transport problem.

Figure 1, shows a diagram of a transport problem, transporting between different points of origin (am) and destination (bn), where Cij denotes the unit cost from the origin to the destination j, and Xij the transport quantity from the origin and to destination j.

A mathematical model of a closed transport problem

The transport problem is the problem of the linear programming of the m + n equations with m · n variables. The system contains m + n-1 independent equations, which implies that the solution must contain m + n-1 variables. A solution with less than m + n-1 variable values is degenerate. In order for the transport model to be properly represented by a mathematical model, it must be formulated, that is, mathematically set. The function of the target of the decision variable of that constraint needs to be specified. The mathematical model is represented by the transport problem between the origin and the destination j, as shown in Figure 1. Given that the closed system, demand is then equal to supply, which could be mathematically written as follows:

$$\sum_i a_i = \sum_j b_j \quad 1$$

Setting up a transport problem:

- Total obtained from the point of origin and is a_i, where i = 1,2, ..., m (m-number of points of origin)
- The total demand from destination j is b_j where j = 1,2, ..., n (destination n-number)
- c_{ij} = the cost of transporting a unit of goods from its origin and to its destination j, for i = 1,2, ..., m, and j = 1,2, ..., n.
- X_{ij} = quantity of goods to be translated from the origin and to the destination j for i = 1,2, ..., m, and j = 1,2, ..., n.

Goal Function:

- min Z = $\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij}$, the min prefix is in front of the function because the minimum total transport cost is required.

Source restriction:

- $x_{11} + x_{22} + \cdots + x_{1n} = a_1$
- $x_{m1} + x_{m2} + \cdots + x_{mn} = a_m$
- $\sum_{j=1}^{n} = a_i$, i=1,2,...,m - the sum of demand for goods at destination j is equal to the supply of origin i.

Destination limit:

- $x_{11} + x_{22} + \cdots + x_{m1} = b_1$
- $x_{1n} + x_{2n} + \cdots + x_{mn} = b_n$
- $\sum_{i=1}^{m} X_{ij} = b_j$, j=1,2,...,m - the sum of the supply of goods at the origin i, is equal to the demand of the destination j.

Overview of mathematical model:

- min Z = $\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij}$ - objective function
- $\sum_{j=1}^{n} = a_i$, i=1,2,...,m - restriction
- $\sum_{i=1}^{m} X_{ij} = b_j$, j=1,2,...,m - restriction
- $X_{ij} \geq 0, za\ i = 1,2,\ldots,m\ i\ j = 1,2,\ldots,n$- decision variable.

2.1 Program code in matlab

Input part of the code:

```
% Troubleshooting with maximum flow rate using minimal theory of cost-cutting
Flow = sparse ([1 1 1 2 3 3 4 5 6 6 6 7 7], [2 3 4 7 2 6 5 8 2 5 8 6 8], [4 2 3 2 3 1 2 1 4 3 4 2 3 ], 8.8);
```

```
[M, F, K] = graphmaxflow
(Flow, 1.8);
% M is the maximum throughput
of the product
% F is the flow on each line
% K is the minimum cut (result shown in
matrix)
view(biograph(F,[],'ShowWeig
hts','on')) % Broj prikaza
set(h,Cvorova(K(1,:)),'Color
',[1 0 0]);
```

The output, that is, the approximation of the solution is shown in the figure:

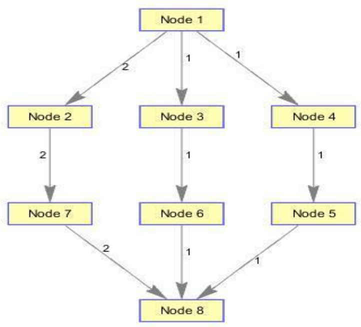

Figure 2. Outline of the path between the transport node with the maximum flow rate and the minimum cost.

2.2 Example of a mathematical model of a closed transport problem

Suppose a particular chain of stores has 3 warehouses and 4 showrooms. Warehouses (starting points) will be marked with I1, I2, I3, while exhibition spaces (destinations) will be marked with O1, O2, O3, O4. If warehouses have goods to fill 2, 6 and 7 trucks a day and showrooms can sell 3, 3, 4 and 5 trucks a day, it is necessary to create a driving plan so that the goods are transported from the starting point to the destination as soon as possible time. The time required for a particular truck to move from its starting point to its destination is expressed in minutes and is given in the table. [7]

Destinations / Originations	O_1	O_2	O_3	O_4
I_1	20	11	15	13
I_2	17	14	12	13
I_3	15	12	18	18

Table 1. Time required for the truck to move from its origin to its destination [7]

It is now necessary to set up a mathematical model for this transport problem, which will help to solve the problem mathematically, with the help of various methods of solving transport problems.
If the goods demanded are equal to the amount of goods that can be delivered then this problem is called a closed transport problem.
Let xij be the quantity of goods transported from the warehouse, ie the starting point I_i, to the showroom, ie the destination O_j. Let the transport of goods from origin I_1, which is the capacity of 2 trucks, to destination O_1 take 20 minutes, while transport from origin I_2 to destination O_2 takes 11 minutes, and so on. For the sake of clarity, a transport table is provided.

Destinations / Originations	O_1	O_2	O_3	O_4	Number of available trucks
I_1	20 x_{11}	11 x_{12}	15 x_{13}	13 x_{14}	**2**
I_2	17 x_{21}	14 x_{22}	12 x_{23}	13 x_{24}	**6**
I_3	15 x_{31}	12 x_{32}	18 x_{33}	18 x_{34}	**7**

Broj potrebnih kamiona	**3**	**3**	**4**	**5**	**15 15**

Table 2: Transport table [7]

The transport problem is a linear programming problem (unit times are linear with respect to the number of trucks) and can be solved in many ways:

- simplex method or
- special methods for solving linear transport problems.

If we solved the simplex method, then the mathematical model for the given problem would look like this:

$$Z = 20x_{11} + 11x_{12} + 15x_{13} + 13x_{14} + 17x_{21} +$$

$$14x_{22} + 12x_{23} + 13x_{24} + 15x_{31} + 12x_{32} + 18x_{33} + 18x_{34}$$

With restrictions:

$$x_{11} + x_{12} + x_{13} + x_{14} = 2$$

$$x_{21} + x_{22} + x_{23} + x_{24} = 6$$

$$x_{31} + x_{32} + x_{33} + x_{34} = 7$$

$$x_{11} + x_{21} + x_{31} = 3$$

$$x_{12} + x_{22} + x_{32} = 3$$

$$x_{13} + x_{23} + x_{33} = 4$$

$$x_{14} + x_{24} + x_{34} = 5$$

We can solve this transport problem by special methods of solving transport problems. These methods can be classified into two categories:

- methods for determining the initial (basic) solution,
- methods for obtaining the optimal solution (created by improving the initial solution, ie the basic solution).

The methods for determining the initial (basic) solution include:

- north-west corner method
- least cost method
- Vogel's method

The methods for determining the optimal solution include:

- Stepping Stone Method
- Odds method or modified distribution method (MODI)
- Deployment method

The above methods for determining the optimal solution first check whether the initial basic solution is optimal or not. If the initial basic solution is not optimal, each of the methods is shown to move to a better basic solution, ie a basic solution that ensures the reduction of the total transportation costs. Thus, by solving the northwest corner method, we obtained the initial basic solution, ie the transport time is Z = 249 minutes. With the lowest cost method, the transit time is Z = 207 minutes, while Vogel's Z = 199 minutes. With the method of jumping from stone to stone the initial basic solution, ie time, transport Z = 249 minutes was improved to Z = 199 minutes. Also, with the MODI method we improved the initial basic transport time of Z = 249 min to Z = 199 minutes.

3. Concluding considerations

This paper presents a solution to the transport problem through a mathematical model. So, as we have seen, we have defined the input variables, constraints, and mathematical function. We have set constraints on the origin, destination. We then defined the objective function and then created a mathematical model. In a software solution in matlab in a couple of lines of code, we have presented a solution to the problem with maximum throughput and minimum cutting cost. The exit was Nodes or knots, and the trajectory of the minimum cost of transport costs. Also, by solving the example of a closed transport problem, we have shown the results of several methods for solving transport problems.

1. Literature

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