RELATIONSHIP BETWEEN TRANSITION CURVES (CLOTHOIDS) AND CIRCULAR CURVES IN ROAD ALIGNMENT

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Summary

The transition from a straight line to a circular curve is always abrupt, even if this is not readily noticeable at very large radii or low driving speeds. On the straight section the steering wheel angle is zero, on the arc section the steering wheel angle must have a certain value, as stated above, which is greater the tighter the curve. This in turn would mean that you would have to turn the steering wheel jerkily at the transition between the straight section and the circular curve section. Otherwise you would go off track. Even in the case of large radii without transition curves, the lines of a road appear inelastic and can satisfy neither aesthetically nor driving psychologically down to the last detail. In perspective foreshortening - how the road is seen by the driver - sudden kinks appear despite the large curve radii.

Keywords: *clothoid, curve radius, road, road alignment, road safety, transition curve*

1 INTRODUCTION

1.1 Basic principles of road alignment

The alignment of a road is the definition of the route according to direction and inclination. The decisive factors are the type of road (road significance) and the existing terrain. Routing is a technical and design task. The route must be harmoniously coordinated in the site plan and longitudinal section and its good integration into the terrain must be aimed for. The spatial effect on the road user after the construction of the route must therefore always be taken into account. Driving dynamics, traffic safety and aesthetics demand a continuity of the alignment in order to enable a psychologically consistent sensation and a balanced driving style. The course of the road must be unambiguous from the driver's perspective. A traffic route design is shown in three different plans (= different drawing levels) [1]:

- Site plan (= layout),
- Longitudinal section, and
- Cross section (= profiles).

1.1.1 Principles for the use of straight lines

Since the straight line is the direct and thus shortest connection between two points, it is the historical ideal in terms of road layout. In today's road design, longer straight lines with a constant longitudinal gradient should be avoided for roads outside built-up areas. They make it difficult to estimate distances, which has a particularly negative effect on overtaking, they tire the driver, at night there is a risk of glare from oncoming traffic and the fit into the terrain is usually unsatisfactory. However, it can also be advantageous to route with straights, e.g.:

- In plains and wide valleys, especially for routing parallel to existing railway lines, canals or the like,
- If, for traffic engineering reasons, routes with a large visibility range are required for unbundling procedures or for the clear layout of junctions, and
- For the creation of overtaking sight distances on two-lane roads of EKL 3.
- In these cases, the length of straights should always be limited to a value that is still reasonable for the driver. According to RAL [2], the empirical value is usually max L_G = 1,500 m [3].

For safety reasons, the length of straight lines and subsequent circular curves should be in a balanced ratio. This requires both a minimum length of straight lines between circular curves and a minimum radius following straight lines of a certain length. In the following illustration (Fig.1) of the permissible radii following straight lines according to RAL [2], only the good range with $R/L_G > 1.5$ or $R > 450$ m is provided as a rule for the design classes EKL 1 to EKL 3 [3].

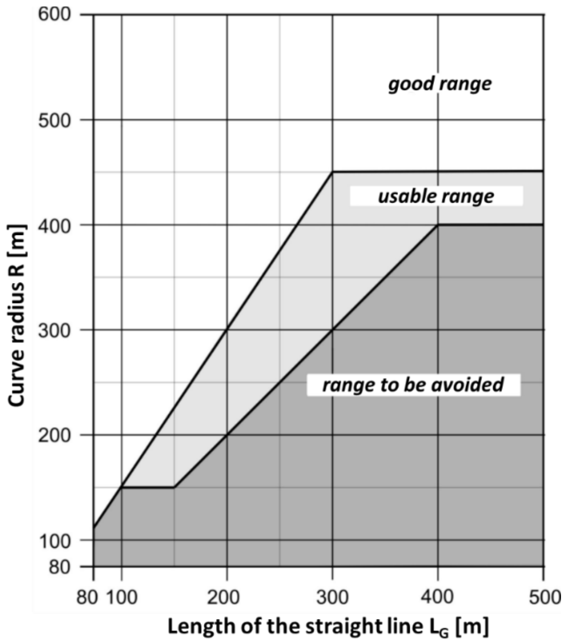

Figure 1. Permissible radii following a straight line [2, 3] (edited by author)

Straight lines between bends curved in the same direction should be avoided; instead, a larger continuous circular curve should be used, for example. If this is not possible, their length should be at least 600 m for roads in EKL 1 to EKL 3 and at least 400 m for roads in EKL 4 [3].

According to the RAL [2], the cross-section design is a defining feature for the principle of standardised and recognisable road types. A one-lane standard cross-section and a special form of marking are provided for each design class (Fig.2) [4].

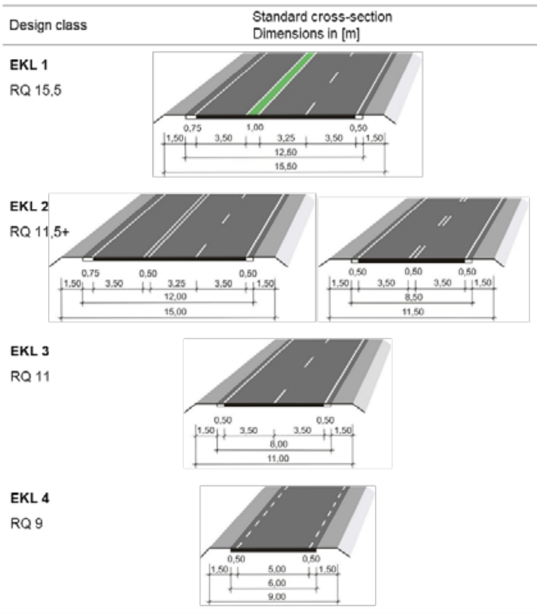

Figure 2. One-way standard cross-sections in accordance with the RAL [4] (edited by author)

1.1.2 Principles for the application of circular curves

The circular curve is the most commonly used alignment element. It is characterised by its radius and the arc length utilised for the road axis. The radii used and their consequences should enable smooth driving at a speed appropriate to the design class and be in harmony with the topography. In addition to the lower limits of this radius size, which are necessary from the point of view of driving dynamics, the RAL [2] also specify upper limits of the recommended radii. These are intended to ensure that both excessively stretched line layouts in the design classes EKL 3 and EKL 4 with possibly suggested overtaking possibilities are avoided and that the intended overall effect of the different road types and their distinguishability is maintained.

The recommended radii (Table 1) can be undercut by a maximum of 15% for roads in EKL 2 to EKL 4 in exceptional cases that must be justified. The prerequisite for this, however, is that the ratios of successive radii are then in the good range. The minimum lengths shown guarantee a minimum passage time and thus the perceptibility as an independent element [3].

Table 1. Recommended radii and minimum lengths of circular curves [2, 3] (edited by author)

Design class	Radius ranges R [m]	Minimum lengths of circular curves min L [m]
EKL 1	≥ 500	70
EKL 2	400 – 900	60
EKL 3	300 – 600	50
EKL 4	200 – 400	40

A special role is played by the balanced ratio of successive circular curves, which helps to avoid significantly different speed levels in successive curves. In design classes EKL 1, EKL 2 and EKL 3 the good range is decisive; for roads in design class EKL 4 the usable range is sufficient (Fig.3) [3].

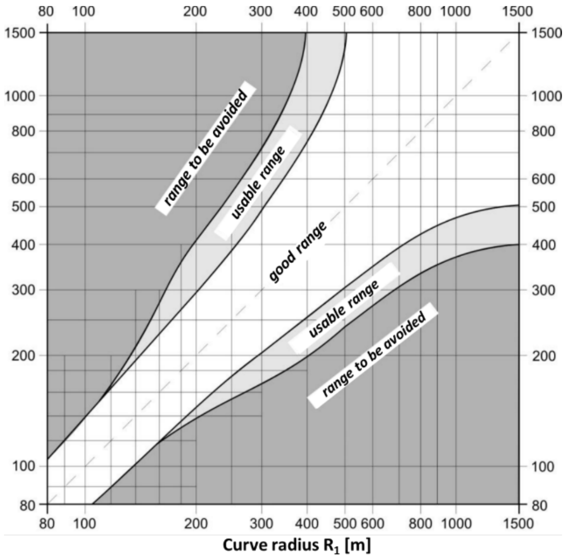

Figure 3. Ratio of successive radii [2, 3] (edited by author)

Short curves should be avoided as far as possible:

- Successive short curves tempt to "drive straight through"; replace with straight line or large curve;
- Short arcs with a small radius appear as a kink (applies in the site plan as in the longitudinal section); replace by a large radius with a longer curve length.
- Use small radii in good coordination with the radii of adjacent circular curves (relation alignment):
- In the case of small radii between large radii or after straight lines, unexpected curve discontinuity occurs with increased risk of accidents; replace with a larger radius or implement safety-improving measures.
- Curve discontinuity is present with subsequent similarly small/large radii: successive radii should be coordinated with each other in order not to cause large speed differences (Fig.4) [1].

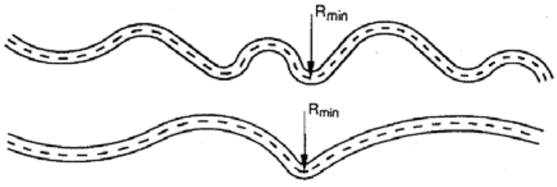

Figure 4. Principle of a curvilinear (top) and non-curvilinear (bottom) alignment [1]

The curvature k = 1 / R in the circular curve is constant, since the radius in the circular curve is also constant. This results in the following curvature band for a right-hand curve (Fig.5):

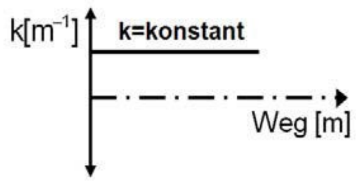

Figure 5. Curvature diagram of the circular curve (right-hand curve) [1]

The design speed (V_P) is the basis for determining the minimum lengths of the clotoids and circular curves, the fillets in the longitudinal section (crest and trough radii) and the minimum values for the required sight distances. As a rule, the line layout is to be selected in such a way that the design speed does not change abruptly. For motorways, the design elements of location and height are to be selected in such a way that they do not result in a lower design speed than 100 km/h [1]. The relationship between radius (R), longitudinal slope s and speed (V_{85}) for freely moving passenger cars according to the Guideline for Planning, Construction and Maintenance of Roads RVS 03.03.23 [5] is shown in Table 2:

Table 2. Relationship between radius (R), longitudinal slope s and speed (V_{85}) for a freely moving passenger car [5]

V_{85}	[km/h]	40	50	60	70	80	90	100	110	120	130
R	[m]	30	50	80	130	200	300	400	500	600	800
s	[%]	12	11	10	9	8	7	6	5	4,5	4

1.1.3 Principles for the use of clothoids

Due to driving dynamic, drainage, visual, driving psychological and aesthetic considerations, a transition curve is used when the curvature is changed. The transition curve has three tasks to fulfil [1]:

- It is intended to provide a steady change in the centrifugal acceleration occurring during cornering when transitioning from one curvature to another.

- It is used as a transition section for roadway twisting (change of cross slope).
- It ensures a fluid line course through a gradual change in curvature and thus serves a visually satisfactory alignment (Fig.6).

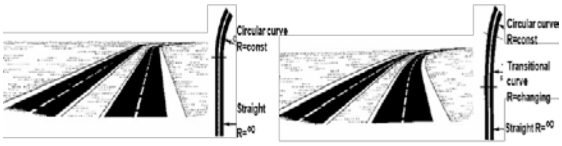

Figure 6. Route without (left) and with (right) transition curve [1] (edited by author)

In general, the following requirements are to be placed on a function that is to be used as a transition curve [1]:

1. Continuous change of the curvature, preferably proportional to the length;
2. A range as long as possible in which it can be used as a transition arc;
3. Points of inflection with place R = ∞ or curvature = 0 for connection to a straight line;
4. Simple mathematical handling.

In road and railway construction, only the clothoid is used as a transition curve, as it optimally fulfils the first three of the requirements. The so-called preliminary arch as a transition curve is no longer provided for in the Austrian guidelines. The clothoid is used as a transition curve in road and railway construction. The clothoid is a spiral curve with steadily increasing curvature, for which the product of the curve length L and the associated radius of curvature R is constant [1]:

$$A^2 = L * R$$

R ... Circular curve radius [m]

L ... Length of the transition curve (clothoid) [m]

A ... Parameters of the clothoid [m] (Fig.7).

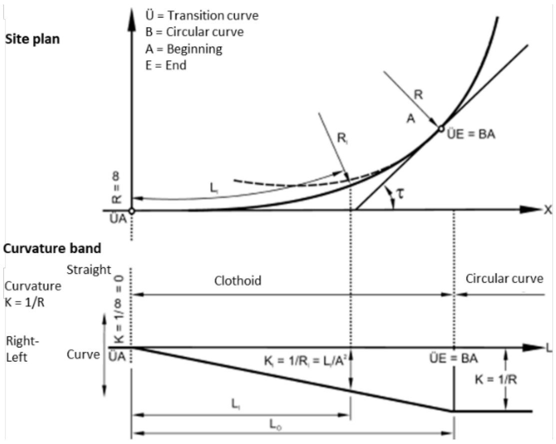

Figure 7. Design elements in the site plan: Curve and clothoid [6] (edited by author)

In principle, a radius as large as possible should be chosen in the routing. In any case, certain minimum radii are to be maintained according to the design/project planning speed (risk of skidding). Furthermore, the ratio of successive radii should be in the range of 1:1 to 1:3. The total curvature (sum of the tangent angles) should also not be too large, otherwise the design speed cannot be maintained. If, on the other hand, the curvature is far above this, the roadway must be made narrower so as not to tempt users to speed.

Since the curvature of the clothoid changes linearly with the length of the curve, this has the effect for the driver that he must turn his steering wheel at a constant angular velocity at constant speed in order to drive the traced transition curve [4]. The clothoid parameters should lie in the following range:

$$R \geq A \geq R/3$$

With the clothoid, a steady curvature progression is made possible when transitioning from one curvature to another or straight lines to a circle. The curvature band of the clothoid as well as the most important formulas are shown in Fig.8.

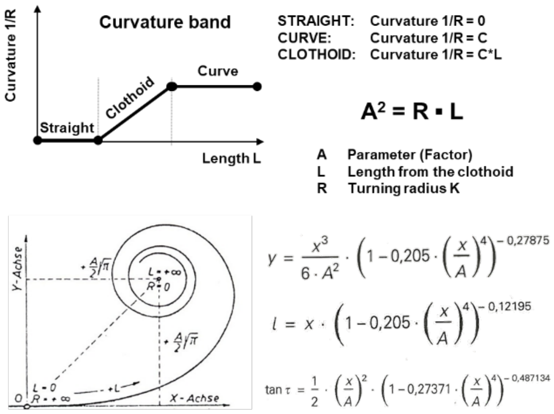

Figure 8: Curvature band of the clothoid and the most important formulas [7] (edited by author)

The minimum length of the clothoid L_{min} depending on the design speed according to the Guideline for Planning, Construction and Maintenance of Roads RVS 03.03.23 [5] is shown in Table 3.

Table 3. Minimum length of the clothoid L_{min} depending on the projecting speed [5]

V_P (km/h)	40	50	60	70	80	90	100	110	120	130
L_{min} (m)	15	20	30	39	44	50	56	61	67	72

1.2 The connection between curve radius and accident occurrence on roads

The correlation between curve radius and accident occurrence was investigated by Krebs/Klöckner [8] and Leutzbach/Zoellmer [9]. As the following graph shows (Fig.9), the accident rate decreases with increasing radius [10].

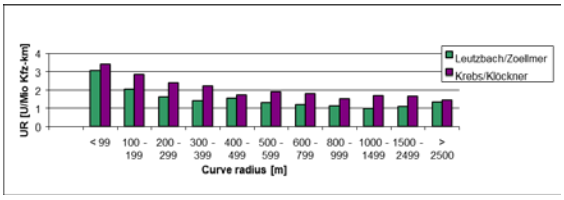

Figure 9. Accident rates (UR = germ. Unfallraten) as a function of curve radius [10] (edited by author)

Furthermore, Leutzbach/Zoellmer [8] investigated the element sequences straight line - circular curve or straight line - clothoid - circular curve. The results showed a clear increase in safety with increasing radius up to a size of R = 200 m. Above this threshold, no systematic correlations could be detected. Above this threshold, no more systematic correlations could be detected. In the range up to R = 200 m, the sequence "clothoid before radius" showed lower values in the accident characteristics than the sequence "straight line before radius". This thesis illustrates the contribution of clothoids to road safety, especially for small radii. [4].

The radii obtained by the Geometry Classifier programme are divided into classes of 50 metres. Radii greater than or equal to 1,000 metres are combined in one class, since only 4 UPS (germ. *Personenschadenunfälle*) (personal injury accidents) occurred in the radius range of 1,000 to 2,000 metres. Figure 10 shows the distribution of accidents among the radius classes, the class width is 50 m [10]. UPS = "Accidents with personal damage" Accidents with personal damage.

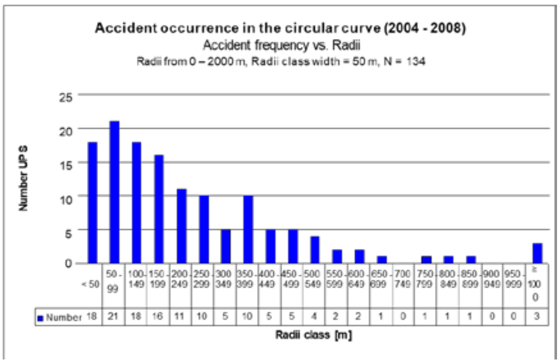

Figure 10. Accident frequency by radius, class width 50 m [10] (edited by author)

The highest averaged accident rate and the highest relative accident risk occur at radii of 50 to 99 m. Both accident indicators show a clear trend. With increasing radius, both accident indicators decrease, any deviations can be considered as outliers. This confirms the statement from the literature research that the risk of accidents decreases with increasing radius. The study shows that smaller radii have a greater accident potential than larger ones. As the literature research has shown, it is not only the individual radius that is responsible for the accident, but also the coordination of the radii in the course of the road. Already during the planning phase, care should be taken to ensure that the alignment elements are coordinated with each other and that there is a good view of the route [10].

The guideline values of Lippold [11] can be found here as the boundary between the usable range and the range to be avoided. The transition to the good range is 50 % above the usable range (150 m $\leq L_G \leq$ 300 m). From straight line lengths of L_G = 300 m, radii of R ≥ 450 m are to be provided [4].

In the course of the investigations [12], suitable stretches of road with standards similar to those of EKA 2 according to RAA [13] were selected in the existing network of motorway-like federal roads in the extended Stuttgart metropolitan area. The existing curve radii were evaluated according to their frequency of occurrence and their accident risk. The investigation focused on curves with radii R ≤ 1000 m, as these had been shown to have a higher accident risk according to previous research.

The investigation of the routing in the site plan was carried out using the road data stored in the road database (TT-SIB/Infosys). The frequency of circular curves with radii R ≤ 1000 m was evaluated and, in the case of circular curves with an identified higher accident risk, also the relation of the radii of successive circular curves and following a long straight line. The study included 173 km of motorway-like federal roads. These have a total of 213 circular curves. Of these, 72 circular curves have radii with R < 1000 m (33.8%), for which an increased accident risk can be assumed according to [1]. Circular curves with small radii with R ≤ 750m are represented by 33 (15.5%), including 16 circular curves with R < 650 m (7.5%). All circular curves with radii R ≤ 650m were assessed qualitatively according to their statistical accident risk as well as with the help of the electronic accident maps and video images [12].

The investigations showed that circular curves with radii R < 650 m, especially in left-hand curves, always have accident clusters and statistically have about twice the accident risk than radii with R ≥ 650 m, despite sometimes drastic speed restrictions. This also tends to apply if the radius ratio R1 / R2 ≤ 1.5 is observed [12].

1.2.1 What exactly happens when driving at high speed into the curve with radius without clothoid after long straight line?

The transition curve is an alignment element used in the construction of traffic routes as a connection between a straight line and a circular curve or between two circular curves. It is characterised by the fact that it has a different radius of curvature at each point, like an ellipse and in contrast to a straight line and a circular curve. This has the effect that in the transition between a straight line and a circular curve, the curve narrows only gradually and not in leaps and bounds.

On a road without a transition curve, the steering wheel would have to be turned in jerkily (suddenly) when going from a straight lane into a curve. Even if, despite the abrupt steering movement, the car stays in the lane (which is extremely difficult to do even on a dry road!), the driver is jerked - he (and the car) experiences a jolt! Transition curves, on the other hand, can be used to steer slowly from the centre position to the necessary steering wheel turn. The purpose of a transition curve is to continuously build up or reduce the centrifugal forces occurring in the curve when driving through it, and not to suddenly.

Without a transition curve, the acceleration forces on the passengers and the construction would also be too high when entering the loop of a roller coaster. Let's assume that the driver has somehow managed to enter a smaller circular curve (quarter circle) with his car at high speed. Since its curvature k is constant, the driver can keep the steering wheel in a fixed position if he drives optimally. The centrifugal force pushes the driver outwards. At the end of the quarter circle, there is suddenly a straight line with zero curvature. The driver must now abruptly bring the steering wheel into the straight-ahead position. Even if the car stays in the lane despite the abrupt steering movement (which is extremely difficult to do even on a dry road!), the driver is jerkily moved back to his normal sitting position (or beyond) - he (and the car) experiences a jolt.

1.2.2 Catastrophic accident on the motorway-like route with small radius after long straight line

On 23 November 2021, this terrible accident with 46 deaths occurred on the motorway near the village of Bosnek in western Bulgaria. There are no words to describe this horror, especially because the children were also victims. Looking at this photo of the accident site alone, one immediately notices that the curves of this motorway route were probably not projected with transition curves (clothoids), which is a must, especially as possibly here with the most unfortunate combination of small radius curve after long straight. The curves with smaller radius and without clothoids are visually perceived as kinks (Fig.11).

Figure 11. The terrible accident with 46 dead on the motorway near the village of Bosnek in western Bulgaria [14]

How a curve with the clothoid looks visually, especially important for direct comparison with fig.11, can be seen in fig.12.

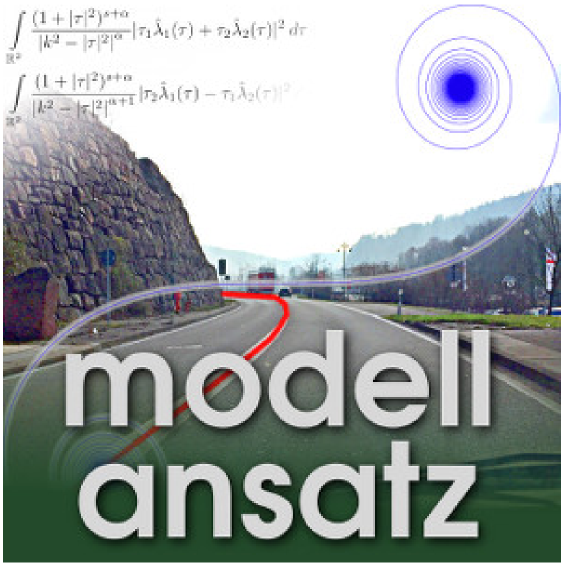

Figure 12. A route with the clothoid [15]

According to European Commission data, Bulgaria had the second highest rate of road deaths in the European Union in 2019, with 89 fatalities per million inhabitants. "*We hope that we can learn our lessons from this tragic incident and prevent such incidents in the future*" said Bulgarian caretaker Prime Minister Stefan Yanev, who also visited the scene of the accident [14].

The following photo (Fig.13) shows a very significant example of upper designs of road alignment: the left bridge is designed in a long straight line, where the curve comes immediately after the end of the bridge. The middle, higher bridge, on the other hand, is projected in its entire length as well as in the further course of the motorway with clothoids and large curve radii. The bridge on the far right is still under construction and represents a railway bridge.

Figure 13. The construction site of the Nahuai River Bridge on the Panzhou-Xingyi High-speed Railway [16]

CONCLUSION

A circular curve has a constant curvature. This means that its representation lies parallel above the abscissa axis. A straight line also has a constant curvature, for which the radius is infinite and the curvature (in contrast to the circular curve) is therefore zero. Thus the representation of the curvature coincides with the abscissa axis. If a straight line is tangentially connected to a circle, the curvature at the point of contact takes on two different values, depending on the side from which this point is approached. This results in a jump in the curvature pattern. For the driver, this would mean that he/she would have to turn the steering wheel in time t = 0 so that the wheels are steered at this point according to the changed curve, which is unrealistic. In addition, the full centrifugal force acts on the vehicle occupants at this point. Therefore, a gradual transition from driving in a straight line to driving in a circular curve must be designed. This is made possible by the use of a clothoid between the straight line and the circular curve in such a way that the curvature or the radius of the circle changes linearly with the path length.

Checking the routing is an essential step in order to avoid later accident black spots or new planning apriori. A small radius in the horizontal curve, especially after a long straight line, represents a very unfavourable routing and element selection with an "optical kink" is marked. Even though the multitude of requirements for a good alignment may seem complex at first, they are normal for an experienced planner. However, since the effects of good or bad routing are often difficult to explain to clients or third parties or only become apparent later, it is quite common to make the routing results in difficult terrain conditions recognisable with the help of visualisations or simulations

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